A NOTE ON THE COMPLEX POYNTING VECTOR, AND ON THE FRACTIONAL CURRENT ON THE UPPER SURFACE OF A MICROSTRIP LINE

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Summary

It is shown how the two forms $E \times H^*$ and $E^* \times H$ should be used when extracting reactance information from the complex Poynting Vector in a microstrip problem. When the strip is narrow, some of the axial current appears on top of the strip. Its value is calculated and the practical use of this information is indicated.

During the course of the near-field investigation of a microstrip open-circuit discontinuity a number of features of a more general character were encountered, and are reported on here.

An early calculation[1] of radiation from the open-circuit used the far-field Poynting vector method, and assumed that the effective (relative) dielectric constant ε' was to be used in all terms in the calculation. A subsequent calculation[2] using the actual constant ε in the polarization term, also via the far-field Poynting vector method, agrees with a quite different calculation by van der Pauw.[3] A near-field calculation[4] using only the effective constant ε' agrees with the far-field calculation of reference 1. It was therefore rather disturbing to discover that the near-field calculation, using both constants ε and ε', did not at first agree with the results of reference 2. The full details of this near-field calculation, which also yields the reactive features of the discontinuity, will be reported elsewhere in due course. Here we wish to discuss the outcome of the method found to reconcile the differences between the two approaches.

In the near-field calculation the surface used is in the form of a rectangular box that just includes the upper microstrip line. As shown in figure 1, there are the two sides $x = \pm w/2$, $0 < y < t$, $z < 0$; the aperture $z = 0$, $0 < y < t$, $-w/2 < x < w/2$; and the top $-w/2 < x < w/2$, $y = t$, $z < 0$. The lower surface $y = 0$ does not contribute because the tangential electric field is identically zero there. This field should also be zero on the metal at the top of the strip, but in fact is non-zero there due to the assumed (approximate) sinusoidal form of the axial current. (A similar phenomenon is well known for the case of a half-wave dipole antenna). There will therefore be a component of the Poynting vector if there is current, and hence magnetic field, on the top of the strip. As will appear later, this will be associated with an axial component of magnetic field, giving a further contribution from the sides. And finally, under these circumstances, the electric field lines from the top of the strip will terminate on the ground plane outside the strip region, giving rise to polarization sources outside the Poynting vector surface. All three of these additional terms are needed, and they correctly reconcile the near and far field calculations provided the current on top of the strip, and the axial magnetic field at its sides take certain specific values. These values are determined as follows.

Maxwell's equations under the strip, if written in terms of two components H_x and E_y only, are not compatible with propagation with an effective dielectric constant ε'. To be compatible an extra term is needed, and of the two possible components E_z and H_z, only the latter is suitable. The equations become

$$\partial E_y/\partial z = j\omega\mu_o H_x \tag{1}$$

$$\partial H_x/\partial z - \partial H_z/\partial x = j\omega\varepsilon_o\varepsilon E_y \tag{2}$$

Assuming propagation in the z-direction to involve the wavenumber $k_o(\varepsilon')^{\frac{1}{2}}$, and eliminating E_y between (1) and (2) gives

$$\partial H_z/\partial x = jk_o(\varepsilon')^{-\frac{1}{2}}(\varepsilon-\varepsilon')H_x \tag{3}$$

On the assumption that H_x is constant beneath the strip, this equation integrates to

$$H_z = jk_o x(\varepsilon')^{-\frac{1}{2}}(\varepsilon-\varepsilon')H_x \tag{4}$$

In the air above the strip the corresponding equation is obtained by taking $\varepsilon = 1$. The continuity of transverse current flow at the strip edges requires that $H_{za} = H_{z\varepsilon}$ at $x = \pm w/2$, where subscripts a and ε are for air and dielectric respectively. This gives

$$H_{xa} = \frac{\varepsilon'-\varepsilon}{\varepsilon'-1} H_{x\varepsilon} \tag{5}$$

as the connection between the transverse magnetic components above and below the strip. In terms of the currents this gives

$$I_{top} = \alpha I_o \, , \quad \alpha = \frac{\varepsilon-\varepsilon'}{\varepsilon-1} \tag{6}$$

where I_o is the total axial current.

Using equations (4), (5) and (6) gives

$$H_{z\varepsilon} = -\alpha\beta\partial I_o/\partial z, \quad \beta = (\varepsilon'-1)/2\varepsilon' \tag{7}$$

The constants α and β in (6) and (7) are exactly of the form needed to reconcile the near and far field calculations. From Wheeler's paper[5], $\varepsilon' = (\varepsilon+1)/2$ for very narrow strips, giving $\alpha = \frac{1}{2}$ in this case, as might be expected. For wide strips, $\varepsilon' \approx \varepsilon$ and then α, from (6) is small; most of the current flows on the underside of the strip.

The complex Poynting vector is usually taken as $E \times H^*$, and if only the real part is needed, this form does not give rise to any problems. When the reactance is also needed it becomes necessary to distinguish between $E \times H^*$ and $E^* \times H$. In the microstrip problem it turns out that both forms are needed, and the determination of the one or the other hinges on what can be termed primary and secondary fields. The distinction can be clearly seen in the case of a half-wave metallic dipole antenna. A current on the wire is assumed, a tangential electric field is calculated from it, and a complex Poynting vector $E \times H^*$ at the wire surface is found. Here, the current is primary, the calculated field is secondary, and $E \times H^*$ gives the series input impedance of the antenna, both resistive and reactive parts.

In the case of the microstrip the electric field under the strip turns out to be primary, and the consequential axial magnetic field is secondary. The form $E^* \times H$ is needed and gives the shunt impedance of the discontinuity. However, when the extra terms coming from the integration on the top surface are needed, the situation is similar to that of the dipole. The form $E \times H^*$ is required and gives a series impedance, which can then be transformed into an equivalent shunt impedance. When a finite length microstrip is examined in this way the mutual admittance between the two ends appears in the formula in a form which, for large L, is asymptotic to $e^{-jk_o L}/L$ where L is the line length. The phase delay in this term is as expected. However, if <u>all</u> the calculation is done using $E \times H^*$, some of the terms involve a (non-physical) phase advance, confirming the need to carefully distinguish the use of the two similar forms.

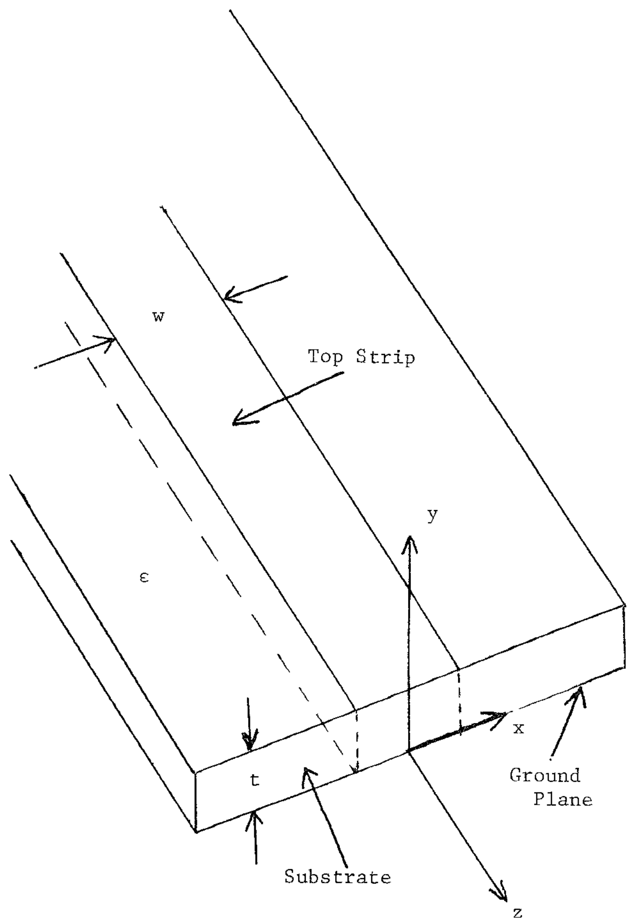

Figure 1. Microstrip Geometry

References

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